

Asynchronous Communication over a Fading Channel and Additive Noise

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Abstract—In [1], Chandar et al studied a problem of sequential frame synchronization for a frame transmitted randomly and uniformly among A slots. For a discrete memory-less channel (DMC), they showed that the frame length N must scale as $e^{\alpha(Q)N} > A$ for the frame detection error to go to zero asymptotically with A . $\alpha(Q)$ is the synchronization threshold and Q is channel transition probability. We study the sequential frame synchronisation problem for a fading channel and additive noise and seek to characterise the effect of fading. For a discrete ON-OFF fading channel (with ON probability p) and additive noise (with channel transition probabilities Q_n), we characterise the synchronisation threshold of the composite channel $\alpha(Q)$ and show that $\alpha(Q) \leq p\alpha(Q_n)$. We then characterize the synchronization threshold for Rayleigh fading and AWGN channel as a function of channel parameters. The asynchronous framework permits a trade-off between sync frame length, N , and channel, Q , to support asynchronism. This allows us to characterize the synchronization threshold with sync frame energy instead of sync frame length.

I. INTRODUCTION

Frame synchronization generally concerns the problem of identifying the sync word, which points the start of a frame, imbedded in a continuous stream of framed data (see e.g., [2]). The problem of detecting and decoding data transmitted sporadically is studied as asynchronous communication. For example, the objective of the asynchronous communication system could be to detect and decode a single frame transmitted at some random and unknown time and there may be no transmission before or after the frame (see e.g., [3]).

The problem of asynchronous communication has been studied earlier in works such as [2] and [4], but the interest has increased in recent times with emerging applications in wireless sensor networks and some control channels. In wireless sensor and actor networks (see e.g., [5] and [6]), the participating nodes would report a measurement or an event to the fusion centre at random epochs. The nodes may need to transmit few bytes of data to the fusion centre over a relatively large time frame, e.g., a single packet possibly in an hour or even in a day. Also, in frameworks such as the Internet of Things [7], the nodes may report measurements sporadically leading to an asynchronous communication framework, however, the constraint on power may be less stringent than in wireless sensor networks. Characterisation of the communication overheads needed in such setup is crucial for optimal network design and operation.

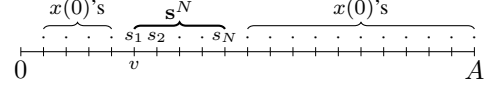


Fig. 1. A discrete-time asynchronous communication model. A sync packet $s^N = (s_1, \dots, s_N)$ is transmitted at some random time $v \sim U\{1, A\}$. The channel input in slots other than $\{v, \dots, v+N-1\}$ is assumed to be $x(0)$.

A. Related Literature

In [1], Chandar et al studied a problem of sequential frame synchronization for a frame transmitted randomly and uniformly in an interval of known size. For a discrete memory-less channel, they defined a synchronisation threshold that characterises the sync frame length needed for error-free frame detection. In our work, we study the sequential frame synchronisation problem for a fading channel and seek to characterise the effect of fading. In [3], a basic framework for communication in an asynchronous set up is proposed and achievable trade-off between reliable communication and asynchronism is discussed. The asynchronous communication set-up is studied in the finite block-length regime in [8]. We restrict our attention to the frame synchronisation problem and study a generalisation that permits us to characterize the scaling needed of the sync frame energy for asymptotic error-free frame synchronisation (instead of the sync frame length N considered in [1] and [3]).

II. PROBLEM SET-UP

The problem set-up is illustrated in Figure 1. We consider discrete-time communication between a transmitter and a receiver over a discrete memory-less channel. The discrete memory-less channel is characterized by finite input and output alphabet sets \mathcal{X} and \mathcal{Y} respectively, and transition probabilities $Q(y|x)$ defined for all $x \in \mathcal{X}$ and $y \in \mathcal{Y}$.

A sync packet $s^N = (s_1, \dots, s_N)$ of length N symbols ($s_i \in \mathcal{X}$ for all $i = 1, \dots, N$) is transmitted at some random time, v , distributed Uniformly in $\{1, 2, \dots, A\}$, where A is assumed known. We suppose that arrival of data from higher layers or occurrence of an event at time v triggers the transmission of the sync packet. The transmission occupies slots $\{v, v+1, \dots, v+N-1\}$ as illustrated in Figure 1, i.e., $x_n = s_{n-v+1}$ for $n \in \{v, \dots, v+N-1\}$, and, we assume that the channel input in slots other than $\{v, v+1, \dots, v+N-1\}$ is $x(0)$ ($x(0) \in \mathcal{X}$ and could represent zero input). The

distribution of the channel output, $\{y_n\}$, conditioned on the random time v and the sync sequence \mathbf{s}^N , is $Q(\cdot|s_{n-v+1})$ for $n \in \{v, v+1, \dots, v+N-1\}$ and $Q(\cdot|x(0))$ otherwise.

The receiver seeks to identify the location of the sync packet v from the channel output $\{y_n\}$. Let \hat{v} be an estimate of v . Then, the error event is represented as $\{\hat{v} \neq v\}$ and the associated probability of error in frame synchronization would be $P(\{\hat{v} \neq v\})$. We are interested in characterizing the sync sequence \mathbf{s}^N needed for asymptotic error-free frame synchronization. In this paper, we assume that the receiver employs a sequential decoder to detect the sync packet. In particular, we assume that the decision $\hat{v} = t$ depends only on the output sequence up to time $t+N-1$, i.e., $\{y_1, \dots, y_t, \dots, y_{t+N-1}\}$.

In [1], Chandar et al define a synchronization threshold that characterizes the sync frame length needed for optimal sequential frame synchronisation for a discrete memoryless channel.

Definition II.1 (from [1]). Let $A = e^{\alpha N}$. An asynchronism exponent α is said to be achievable if there exists a sequence of pairs, sync pattern and sequential decoder (\mathbf{s}^N, \hat{v}) , for all $N \geq 1$, such that

$$P(\{\hat{v} \neq v\}) \rightarrow 0 \text{ as } N \rightarrow \infty$$

The synchronization threshold, denoted as $\alpha(Q)$, is defined as the supremum of the set of achievable asynchronism exponents.

In [1], the synchronization threshold for the discrete memory-less channel was shown to be

$$\alpha(Q) = \max_{x \in \mathcal{X}} D(Q(\cdot|x) \| Q(\cdot|x(0))) \quad (1)$$

where $D(Q(\cdot|x) \| Q(\cdot|x(0)))$ is the Kullback-Leibler distance between $Q(\cdot|x)$ and $Q(\cdot|x(0))$. The authors also provide a construction of sync sequence \mathbf{s}^N entirely with two symbols, $x(0)$ and $x(1)$, where

$$x(1) := \arg \max_{x \in \mathcal{X}} D(Q(\cdot|x) \| Q(\cdot|x(0)))$$

and show asymptotic error-free frame synchronization with a sequential joint typicality decoder¹.

In this work, we study the sequential frame synchronisation problem for a fading channel and seek to characterise the effect of fading. In Section III, we characterise the synchronisation threshold for a composite, general fading and additive noise channel with finite alphabets. The synchronisation threshold for the Rayleigh fading and AWGN channel is studied in Section IV. In Section V, we generalise the frame synchronisation framework and study a tradeoff between sync frame length N and the channel.

III. COMPOSITE FADING AND ADDITIVE NOISE CHANNEL

In this section, we consider a discrete, memory-less, composite fading and additive noise channel $n(h(\cdot)) : \mathcal{X} \rightarrow \mathcal{Y}$ with finite input and output alphabet sets \mathcal{X} and \mathcal{Y} respectively (see Figure 2). The random fading channel $h(\cdot)$ is modelled as $h : \mathcal{X} \rightarrow \mathcal{H}$, where \mathcal{H} is assumed to be a finite alphabet set,

¹See the proof of Theorem V.1 in Section V for details on the joint typicality decoder.

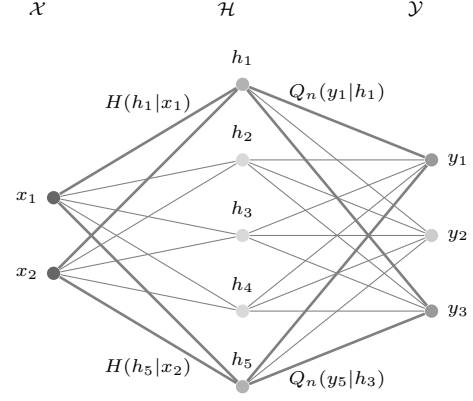


Fig. 2. A composite fading and additive noise channel containing finite alphabets. The transition probabilities for the fading channel and the additive noise channel are provided by $H(\cdot|\cdot)$ and $Q_n(\cdot|\cdot)$ respectively.

with transition probabilities $H(h|x)$ defined for all $x \in \mathcal{X}$ and $h \in \mathcal{H}$. And, the additive noise channel $n(\cdot)$ is modelled as $n : \mathcal{H} \rightarrow \mathcal{Y}$ with transition probabilities $Q_n(y|h)$ defined for all $h \in \mathcal{H}$ and $y \in \mathcal{Y}$. Further, we will assume that the fading process $h(\cdot)$ is independent of the additive noise process $n(\cdot)$. Then, the transition probabilities of the composite channel, $Q(\cdot|\cdot)$, is defined for all $x \in \mathcal{X}$ and $y \in \mathcal{Y}$ as

$$Q(y|x) = \sum_h H(h|x) Q_n(y|h)$$

From Theorem 1 of [1], the synchronisation threshold for the composite fading and additive noise channel is given by

$$\alpha(Q) = \max_{x \in \mathcal{X}} D(Q(\cdot|x) \| Q(\cdot|x(0))) \quad (2)$$

We will now characterise the effect of fading by comparing $\alpha(Q)$ with $\alpha(Q_n)$ (the synchronisation threshold for the additive noise channel). In the remainder of this section, we will restrict to an ON-OFF fading channel, where $h(x) \in \{x, x(0)\}$ for all $x \in \mathcal{X}$. Further, we will assume that the transition probabilities for the ON-OFF fading channel is parameterized by the ON probability p , i.e.,

$$H(h|x) = p I_{\{h=x\}} + (1-p) I_{\{h=x(0)\}} \quad (3)$$

Then, the transition probabilities for the composite channel is given by,

$$Q(y|x) = p Q_n(y|x) + (1-p) Q_n(y|x(0))$$

The synchronisation threshold $\alpha(Q)$ for the composite channel is now characterised in the following lemma.

Lemma III.1. $\alpha(Q) \leq p \alpha(Q_n)$ □

Proof: Define $x(1) := \arg \max_{x \in \mathcal{X}} D(Q(\cdot|x) \| Q(\cdot|x(0)))$.

Then,

$$\begin{aligned} \alpha(Q) &= \max_{x \in \mathcal{X}} D(Q(\cdot|x) \| Q(\cdot|x(0))) \\ &= D(Q(\cdot|x(1)) \| Q(\cdot|x(0))) \\ &= D(p Q_n(\cdot|x(1)) + (1-p) Q_n(\cdot|x(0)) \| Q_n(\cdot|x(0))) \\ &\leq p D(Q_n(\cdot|x(1)) \| Q_n(\cdot|x(0))) \end{aligned} \quad (4)$$

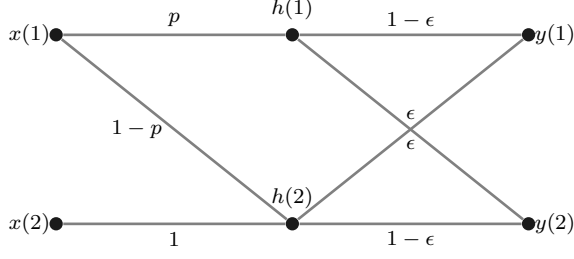


Fig. 3. A composite fading and additive noise channel with binary alphabets. We consider an ON-OFF fading channel with ON probability p and an additive noise channel with symmetric transition probabilities ϵ .

The last equation follows from Jensen's inequality. Also, we know that for an independent fading and additive noise model,

$$\begin{aligned} & \arg \max_{x \in \mathcal{X}} D(Q(\cdot|x) \| Q(\cdot|x(0))) \\ &= x(1) \\ &= \arg \max_{x \in \mathcal{X}} D(Q_n(\cdot|x) \| Q_n(\cdot|x(0))) \end{aligned}$$

Thus, we have, $D(Q_n(\cdot|x(1)) \| Q_n(\cdot|x(0))) = \alpha(Q_n)$ and, substituting in Equation (4), we get,

$$\alpha(Q) \leq p \alpha(Q_n)$$

■

Remarks III.1.

- 1) We note that the sync frame length needed for frame synchronisation increases with channel fading (with OFF probability $1-p$).
- 2) In Figure 3, we have illustrated a composite ON-OFF fading and additive noise channel with binary alphabets. The transition probabilities $Q_n(\cdot|\cdot)$ for the additive noise channel is

$$Q_n = \begin{bmatrix} 1-\epsilon & \epsilon \\ \epsilon & 1-\epsilon \end{bmatrix}$$

and the corresponding synchronisation threshold $\alpha(Q_n)$ is

$$\alpha(Q_n) = (1-\epsilon) \log \left(\frac{1-\epsilon}{\epsilon} \right) + \epsilon \log \left(\frac{\epsilon}{1-\epsilon} \right)$$

The transition probabilities $Q(\cdot|\cdot)$ for the composite channel is

$$Q = \begin{bmatrix} 1-\epsilon & \epsilon \\ p\epsilon + (1-p)(1-\epsilon) & p(1-\epsilon) + (1-p)\epsilon \end{bmatrix}$$

and the synchronisation threshold $\alpha(Q)$ is given by

$$\alpha(Q) = (1-\epsilon_p) \log \left(\frac{1-\epsilon_p}{\epsilon} \right) + \epsilon_p \log \left(\frac{\epsilon_p}{1-\epsilon} \right)$$

where $\epsilon_p = (1-p)(1-\epsilon) + p\epsilon$. Taking $\epsilon \rightarrow 0$, we see that,

$$Q \approx \begin{bmatrix} 1-\epsilon & \epsilon \\ 1-p & p \end{bmatrix} \quad \text{and} \quad \lim_{\epsilon \rightarrow 0} \frac{\alpha(Q)}{\alpha(Q_n)} = p$$

i.e., the bound in Lemma III.1 is tight.

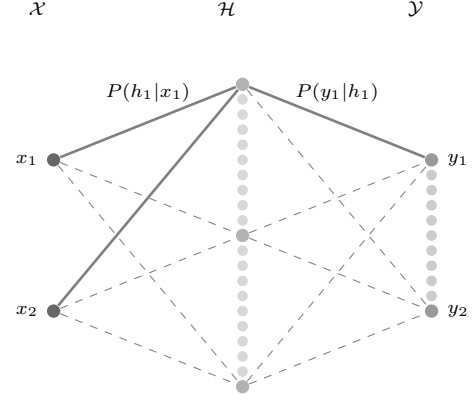


Fig. 4. A representation of Rayleigh fading and additive noise channel with infinite alphabets for \mathcal{H} and \mathcal{Y} .

IV. RAYLEIGH FADING AND AWGN CHANNEL

In this section, we characterise the synchronisation threshold for the Rayleigh fading and AWGN channel. We make the following assumptions about the channel.

- 1) The received signal y_n in slot n is modelled as $y_n = h_n x_n + n_n$ for all n , where h_n is Rayleigh (with scale parameter σ_H) and n_n is WGN with variance σ^2 .
- 2) We assume that the Rayleigh fading and the additive Gaussian noise are independent over slots and independent of each other as well.
- 3) We consider a binary input alphabet set $\{x(0) = 0, x(1) = \sqrt{P}\}$ for the composite channel. P could correspond to the symbol power constraint and $\frac{P}{\sigma^2}$ would then be the SNR. We note that it is sufficient to consider the binary alphabet set for the sequential frame synchronisation problem (see Section II or [1] for details).
- 4) We consider a continuous alphabet set $(-\infty, \infty)$ for the wireless channel (due to the Rayleigh fading and the additive Gaussian noise). The framework developed in the previous section is limited to channel with a finite alphabet set. We have used a large but finite alphabet set in our simulations and have used a limiting approximation to obtain closed-form expressions for the synchronisation threshold.

From equation (2), we know that

$$\begin{aligned} \alpha(Q) &= \max_{x \in \mathcal{X}} D(Q(\cdot|x) \| Q(\cdot|x(0))) \\ &= D(Q(\cdot|x(1)) \| Q(\cdot|x(0))) \end{aligned}$$

where $x(1) = \sqrt{P}$ and $x(0) = 0$. The conditional density functions that characterizes the channel transition probabilities are

$$Q(y|x(0)) = P(n = y) = \frac{1}{\sqrt{2\pi\sigma^2}} e^{-\frac{y^2}{2\sigma^2}}, -\infty < y < \infty$$

and

$$\begin{aligned} Q(y|x(1)) &= P(h\sqrt{P} + n = y) \\ &= \int_0^\infty \frac{e^{-\frac{(y-h\sqrt{P})^2}{2\sigma^2}}}{\sqrt{2\pi\sigma^2}} h e^{-\frac{h^2}{2\sigma_H^2}} dh, -\infty < y < \infty \end{aligned}$$

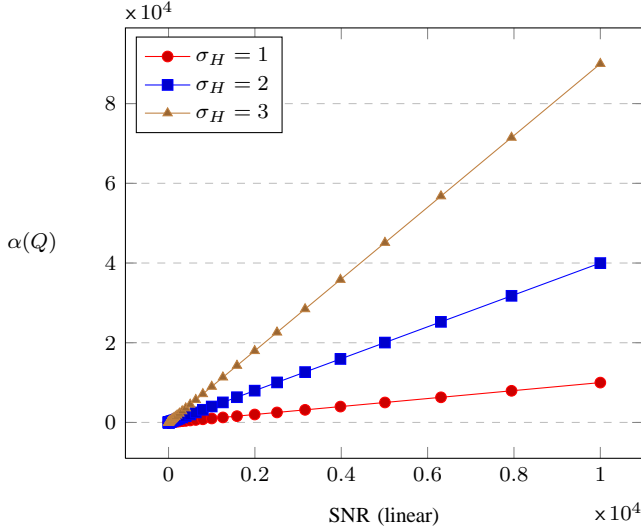


Fig. 5. Plot of synchronisation threshold of the Rayleigh fading channel, $\alpha(Q)$, for different values of σ_H .

For the continuous alphabet set, we can approximate $\alpha(Q)$ as

$$\alpha(Q) = \int_{-\infty}^{\infty} Q(y|x_1) \log \frac{Q(y|x_1)}{Q(y|x_0)} dy$$

We will now characterize $\alpha(Q)$ through numerical evaluation.

Remarks IV.1.

- 1) In Figure 6, we plot the ratio of the synchronisation threshold of the composite channel $\alpha(Q)$ and the synchronisation threshold of the AWGN channel $\alpha(Q_n) = \frac{P}{2\sigma^2}$ (see [1]) as a function of the SNR ($\frac{P}{\sigma^2}$) and for different values of σ_H . From the figure, we note that $\alpha(Q)$ is linear with SNR for large SNR, i.e., $\alpha(Q) \propto \frac{P}{\sigma^2}$ and hence, $\alpha(Q) \propto \alpha(Q_n)$ as well. Also, from Figure 6 (and numerical verification), we observe that,

$$\alpha(Q) \simeq \alpha(Q_n) \cdot 2\sigma_H^2 \quad (5)$$

- 2) We note that the synchronisation threshold $\alpha(Q)$ of the Rayleigh channel can be strictly greater than $\alpha(Q_n)$ (of the AWGN channel) for large values of the Rayleigh scale parameter σ_H . Unlike the ON-OFF fading channel discussed in Section III, the output of the Rayleigh fading channel can have a gain larger than 1 depending on the channel realisation. Equation (5) captures the effect of both the fading channel gain and the channel transition probabilities.

V. A GENERAL FRAMEWORK FOR ASYNCHRONISM

In [1], [3] and even in the previous sections of this paper, we studied synchronisation threshold for the channel that would characterise the sync frame length N needed for error-free frame synchronization. In this section, we will propose a framework that permits a tradeoff between sync frame length N and channel Q to support asynchronism. The framework will allow us to characterise the scaling needed of the sync packet energy for asymptotic error-free frame synchronisation.

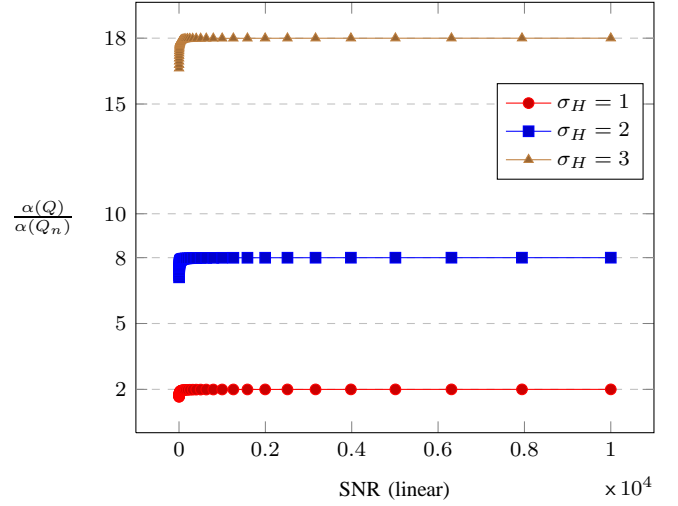


Fig. 6. Ratio of synchronisation thresholds of the Rayleigh fading channel, $\alpha(Q)$, and the AWGN channel, $\alpha(Q_n)$, vs SNR, $\frac{P}{\sigma^2}$.

Consider a sequence of triples, channel, sync word and sequential decoder, $(\{\mathcal{X}_A, \mathcal{Y}_A, Q_A\}, s^{N_A}, \hat{v})$ defined for all $A \geq 1$, where A is the asynchronous interval length. Define $\alpha(Q_A)$ as

$$\alpha(Q_A) = \max_{x \in \mathcal{X}_A} D(Q_A(\cdot|x) \| Q_A(\cdot|x(0)))$$

The following theorem generalizes Theorem 1 in [1] and discusses the necessary scaling needed of N_A and $\alpha(Q_A)$ for asymptotic error-free frame synchronisation.

Theorem V.1. *Consider a sequence of triples, $(\{\mathcal{X}_A, \mathcal{Y}_A, Q_A\}, s^{N_A}, \hat{v})$ parameterized by the period A . Let $N_A \rightarrow \infty$ as $A \rightarrow \infty$ and let $\alpha(Q_A)$ be non-decreasing in A . Then, the probability of frame detection error $P_A(\{\hat{v} \neq v\}) \rightarrow 0$ if $e^{\alpha(Q_A)N_A} > A$.* \square

Remarks V.1.

- 1) Theorem V.1 characterizes the rate at which $\alpha(Q_A) \times N_A$ must scale with A for the frame detection error to tend to zero (asymptotically). In [1], the channel was assumed to be the same independent of N or A . The generalisation proposed in Theorem V.1 enables us to study the tradeoff between $\alpha(Q_A)$ and N_A for supporting asynchronism.
- 2) For an AWGN channel, we know that $\alpha(Q_n) = \frac{P}{2\sigma^2}$. Hence, $\alpha(Q_A)N_A \propto P_A N_A$ (the energy of the sync packet). Thus, the above theorem also characterizes the necessary scaling needed of the energy of the sync packet for the frame detection error to tend to zero.

Here, we have presented only a necessary outline of the proof for Theorem V.1 as the argument is similar to the presentation in [1].

Proof: We consider the framework presented in Section II for every A . A sync packet s^{N_A} of length N_A is transmitted at some random time $v \sim U\{1, A\}$. The discrete memory-less channel is characterised by finite input and output alphabet sets \mathcal{X}_A and \mathcal{Y}_A respectively, and transition probabilities $Q_A(\cdot|\cdot)$

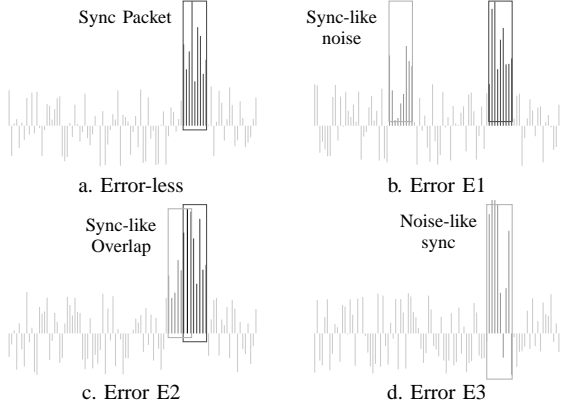


Fig. 7. Examples for each partition of the error event $\{\hat{a} \neq a\}$ during packet reception

with

$$\alpha(Q_A) = \max_{x \in \mathcal{X}_A} D(Q_A(\cdot|x) \| Q_A(\cdot|x(0)))$$

Following [1], we consider a sync sequence \mathbf{s}^{N_A} of length N_A with the following properties.

- 1) Fix some large K , where K is any integer such that $\lfloor \frac{N_A}{K} \rfloor = 2^m - 1$ for some $m = 1, 2, \dots$. Let $s_n = x(1)$ for $\lfloor \frac{N_A}{K} \rfloor < n \leq N_A$. Consider a maximal-length shift register sequence (MLSR) $\{m_n : n = 1, 2, \dots, \lfloor \frac{N_A}{K} \rfloor\}$ of length $\lfloor \frac{N_A}{K} \rfloor$ and map it to $\{s_n : n = 1, 2, \dots, \lfloor \frac{N_A}{K} \rfloor\}$ such that $s_n = x(1)$ if $m_n = 0$ and $s_n = x(0)$ if $m_n = 1$.
- 2) The Hamming distance between the sync sequence \mathbf{s}^{N_A} and any of its shifted sequences is now $\Omega(N_A)$.

We consider a sequential joint typicality decoder for the problem setup. At every time $t + N_A - 1$, the decoder computes the empirical joint distribution \hat{P} induced by the sync pattern \mathbf{s}^{N_A} and the previous N_A output symbols $\{y_t, \dots, y_{t+N_A-1}\}$.

$$\hat{P}_{\mathbf{s}, \mathbf{y}}(x, y) = \frac{N(x, y)}{N_A}, \text{ for all } (x, y) \in \mathcal{X} \times \mathcal{Y}$$

where, $N(x, y)$ denotes the number of joint occurrences of (x, y) in the sync code word and the channel output. The expected joint distribution, P , induced by the sync pattern on the channel output, is defined as

$$P_{\mathbf{s}, \mathbf{y}}(x, y) \triangleq \hat{P}_{\mathbf{s}}(x)Q(y|x)$$

where, $\hat{P}_{\mathbf{s}}(x) = \frac{N(x)}{N_A}$, for all $x \in \mathcal{X}$ with $N(x)$ denoting the number of occurrences of x in the sync code word. If the empirical distribution is close enough to the expected joint distribution, i.e., if $|\hat{P} - P| \leq \mu$ (for some $\mu > 0$), then, the decoder stops and declares $\hat{v} = t$.

The error event $\{\hat{v} \neq v\}$ can be partitioned as discussed below (and as illustrated in the Figure 7).

- $E_1 : \hat{v} \in \{1, \dots, v - N\} \cup \{v + 1, \dots, A\}$. This corresponds to the event that the output symbols generated entirely by the zero input $x(0)$ is jointly typical.
- $E_2 : \hat{v} \in \{v - N + 1, \dots, v - 1\}$. This corresponds to the event that the output symbols generated partially by $x(0)$ and sync word is jointly typical.

- $E_3 : \hat{v} \notin \{v\}$. This corresponds to the event that the output symbols generated by the sync word is not jointly typical.

We note that the event $E_1 \cup E_2$ does not contain the event E_3 as we consider sequential frame detection. Using a union bound, we get,

$$P(\{\hat{v} \neq v\}) \leq P(E_1) + P(E_2) + P(E_3)$$

We will now show that $P(E_1), P(E_2)$ and $P(E_3)$ tend to zero for the listed conditions. Suppose that $A < e^{N_A \alpha(Q_A)}$ for all A . Now, consider an asynchronism length of $\bar{A} = e^{N_A \epsilon_1 (\alpha(Q_A) - \epsilon_2)}$, where $0 < \epsilon_1 < 1$ and $\epsilon_2 > 0$. Clearly, $\bar{A} < e^{N_A \alpha(Q_A)}$.

The probability that the channel output for an input sequence composed entirely of $x(0)$ is jointly typical can be computed as discussed in [1],

$$e^{-N_A(1 - \frac{1}{K})[D(Q_A(\cdot|x(1)) \| Q_A(\cdot|x(0))) + H(Q_A(\cdot|x(1))) - \delta]}$$

where δ is a function of μ (of the joint typicality decoder) and tends to zero as $\mu \rightarrow 0$. Substituting for $D(Q_A(\cdot|x(1)) \| Q_A(\cdot|x(0))) = \alpha(Q_A)$ in the above expression and ignoring the non-negative entropy term, we get the following upper bound

$$e^{-N_A(1 - \frac{1}{K})[\alpha(Q_A) - \delta]}$$

We can now bound $P(E_1)$ using a union bound as,

$$\begin{aligned} P(E_1) &\leq \bar{A} \times e^{-N_A(1 - \frac{1}{K})(\alpha(Q_A) - \delta)} \\ &= e^{N_A \epsilon_1 (\alpha(Q_A) - \epsilon_2)} e^{-N_A(1 - \frac{1}{K})(\alpha(Q_A) - \delta)} \\ &= e^{N_A \alpha(Q_A) (\epsilon_1 - (1 - \frac{1}{K}))} e^{-N_A (\epsilon_1 \epsilon_2 - \delta)} \end{aligned}$$

For large K , small δ (depends on μ of the typicality decoder) and as $A \rightarrow \infty$ (and hence, $N_A \alpha(Q_A) \rightarrow \infty$), we have, $P(E_1) \rightarrow 0$ for any ϵ_1 and ϵ_2 . Thus $P(E_1) \rightarrow 0$ for all $A < e^{N_A \alpha(Q_A)}$.

The probability that the channel output for an input sequence composed partially of $x(0)$ and $x(1)$ (a shifted sequence) is jointly typical can be upper bounded as

$$\begin{aligned} &e^{-N_A \frac{1}{K} [D(Q(\cdot|x(1)) \| Q(\cdot|x(0))) + H(Q(\cdot|x(1))) - \delta]} \\ &\leq e^{-N_A \frac{1}{K} [\alpha(Q_A) - \delta]} \end{aligned}$$

Using a union bound for $P(E_2)$, we get,

$$P(E_2) \leq N_A e^{-N_A \frac{1}{K} [\alpha(Q_A) - \delta]}$$

For small δ and as $A \rightarrow \infty$ (and hence, $N_A \alpha(Q_A) \rightarrow \infty$), we have $P(E_2) \rightarrow 0$.

If $N_A \rightarrow \infty$ as $A \rightarrow \infty$, then $P(E_3) \rightarrow 0$ (follows from the weak law of large numbers).

Thus, we have $P(\{\hat{v} \neq v\}) \rightarrow 0$ if $e^{N_A \alpha(Q_A)} > A$. The above arguments show the achievability of asymptotic error-free frame synchronisation if $e^{\alpha(Q_A) N_A} > A$. The converse follows directly from the discussion in [1], if we consider the special case of $\alpha(Q_A) = \alpha(Q)$ and let $N_A \rightarrow \infty$. ■

The following corollary discusses the application of the Theorem V.1 to an AWGN channel.

Corollary V.1. Consider an AWGN channel with symbol power P_A and noise variance σ^2 . Let $N_A \rightarrow \infty$ as $A \rightarrow \infty$

and let P_A be non-decreasing in A . Then, the probability of frame detection error $P_A(\{\hat{v} \neq v\}) \rightarrow 0$ if $e^{\frac{1}{2\sigma^2} N_A P_A} > A$. \square

Proof: From [1], we know that $\alpha(Q_A) = \frac{P_A}{2\sigma^2}$ for an AWGN channel. From Theorem V.1, we have $P_A(\{\hat{v} \neq v\}) \rightarrow 0$ if $e^{\alpha(Q_A)N_A} = e^{\frac{P_A}{2\sigma^2} N_A} > A$. \blacksquare

Remarks V.2.

- 1) Define $E_A = N_A \times P_A$ as the energy of the sync packet. Then, the above corollary characterises the scaling necessary of the energy of the sync packet for asymptotic error-free frame synchronisation. We note that the synchronisation threshold for the AWGN channel with respect to the sync packet energy is $\frac{1}{2\sigma^2}$.
- 2) A similar result holds for the Rayleigh fading and AWGN channel. The synchronization threshold with respect to the sync packet energy for the composite channel would be $\frac{\sigma_H^2}{\sigma^2}$ (see Figure 6).

VI. CONCLUSION

In this paper, we have studied a sequential frame synchronization problem for a fading channel and additive noise. For an ON-OFF fading channel with ON probability p and an additive noise channel with transition probabilities Q_n , we characterized the synchronization threshold of the composite channel $\alpha(Q)$ and showed that $\alpha(Q) \leq \alpha(Q_n)p$. For a Rayleigh fading and AWGN channel, we characterised the synchronisation threshold as $\alpha(Q) \simeq \alpha(Q_n) 2\sigma_H^2$, where σ_H is the scale parameter of the Rayleigh channel. Finally, we proposed a framework that permits a trade-off between sync word length N and channel Q to support asynchronism. The framework allowed us to characterise the synchronisation threshold for AWGN channel in terms of the sync frame energy (i.e., $e^{\frac{1}{2\sigma^2} E} > A$) instead of the sync frame length N .

The sequential frame synchronisation problem is related to the quickest transient change detection problems studied in works such as [9]. In the future, we seek to generalize the frame synchronisation framework with general definitions for frame synchronisation and the error events.

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